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CORRECTION OF SPHERICAL ABERRATION BY A PHASED LINE SOURCE

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ABSTRACT

Parallel rays incident on a concave spherical reflector cross the axis after reflection. An axial line source can be phased so as to correct for the spherical aberration, thus allowing aberrationless scanning of a pencil beam over at least ±30° in any direction. The phasing of the source varies from end-fire through broadside along its length. Tests made using such feeds as open waveguide, horn, polyrod and corrected slotted and dipole line sources support the theory.

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CORRECTION OF SPHERICAL ABERRATION BY A PHASED LINE SOURCE

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REFLECTORS, SPHERICAL ELECTRONICS (3)
ANTENNAS, SLOT ANTENNAS (9)
ANTENNAS, DIPOLE

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CORRECTION OF SPHERICAL ABERRATION BY A PHASED LINE SOURCE[*] [†]

1. THEORY OF THE SPHERICAL REFLECTOR AND CORRECTED LINE SOURCE

1.1. INTRODUCTION

In contrast to certain optical systems which employ a *point source* with a *corrected optical system*, this paper covers a *line source* so phased that it completely compensates for the spherical aberration of the *spherical reflector*.

It is not necessary to point out that, if the line source problem is solved, we have a perfect solution to the problem of scanning over a sizable solid angle without moving the reflector.[1]

Figure 1(a) shows a section through the center C of such a system. Any ray AP parallel to the line OC, which we shall consider to be the axis of the system, will, after reflection at P, intersect the axis at point F'. By the principle of reversibility, a properly phased line source FF' will reverse the rays, resulting in a plane wave.

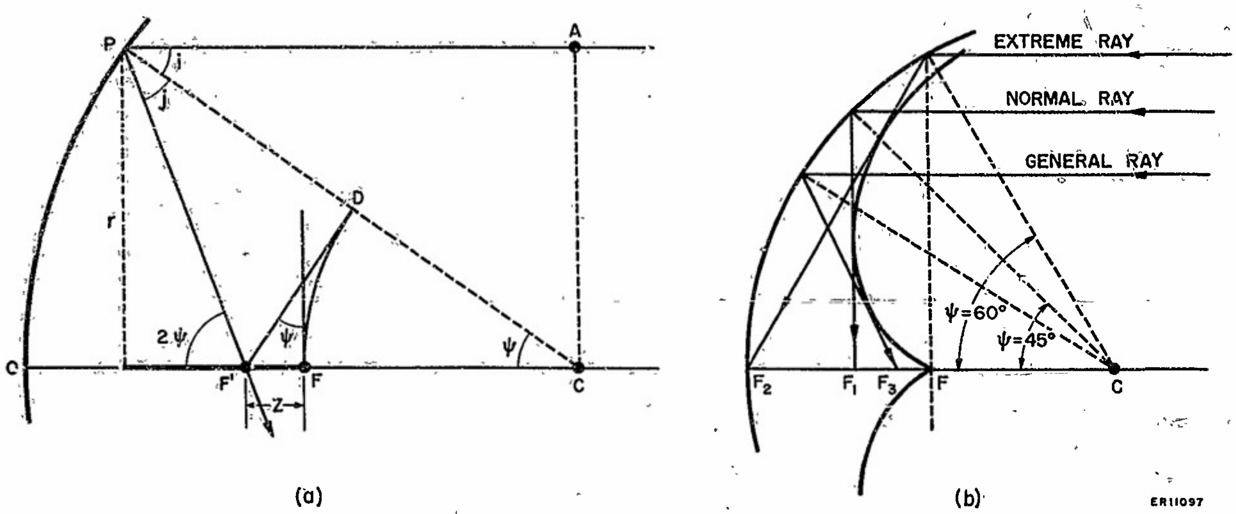

Fig. 1. Geometry of the spherical reflector.

[*] Manuscript received 3 November 1950.

[†] This paper appeared in substantially its present form in *The Proceedings of the National Electronics Conference*, vol. 5, Chicago, Illinois, 1950.

[1] Cf. J. Ashmead and A. B. Pippard, "The Use of Spherical Reflectors as Microwave Scanning Aerials," *J.I.E.E.* vol. 93, Part IIIA, p. 657 (1946) in which is reported an attempt to use a moving combination of lenses and horns to correct spherical aberration. This was abandoned in favor of a point source illuminating a restricted portion of the sphere.

1.2. GRAPHICAL ANALYSIS

The triangle $PF'C$ is isosceles since, according to the law of reflection, $\angle j = \angle i$ and $\angle i = \angle\psi$. The distance CF' is $(R/2)\sec\psi$, which reduces, when ψ is zero, to $CF = R/2 = f$. The distance $FF' = z = f(\sec\psi - 1)$.

Consider that the line source starts at F and extends in the direction FF'. The phasing should be such that the rays emerge at an angle 2ψ with the axis where ψ is given by

$$\sec\psi = 1 + z/f. \tag{1}$$

Thus, a table of secants is sufficient for this information. A graphical value of ψ is obtained by observing the angle between a vertical line at F and the tangent $F'D$ drawn from F' to the circle with center C and radius $R/2 = f$. The maximum value of FF' is FO at which point $\psi = 60°$. When $z = (\sqrt{2} - 1)f$, $\psi = 45°$ and the rays emerge normal to the line source. Various special rays are illustrated in Fig. 1(b), in which the caustic surface, to which all rays are tangent, is also shown.

1.3. PHASE ANALYSIS

The phase at any point may be obtained by noting the path error δ between the reflected ray APF' and the axial ray COF. This is given by

$$\begin{aligned}\delta &= AP + PF' - CO - OF \\ &= R\cos\psi + \frac{R}{2}\sec\psi - R - \frac{R}{2} \\ &= R(\cos\psi - 1) + \frac{R}{2}(\sec\psi - 1)\,.\end{aligned} \tag{2}$$

By use of Eq. (1) this can be put in the form

$$\delta = z\left[1 - \frac{2f}{f+z}\right] = -z\left[\frac{f-z}{f+z}\right]. \tag{3}$$

This is shown in Fig. 2. Note that δ is approximated by $-z(1 - z/f)$ when z is small, and by $-z$ when z is still smaller. (See dashed lines, Fig. 2.)

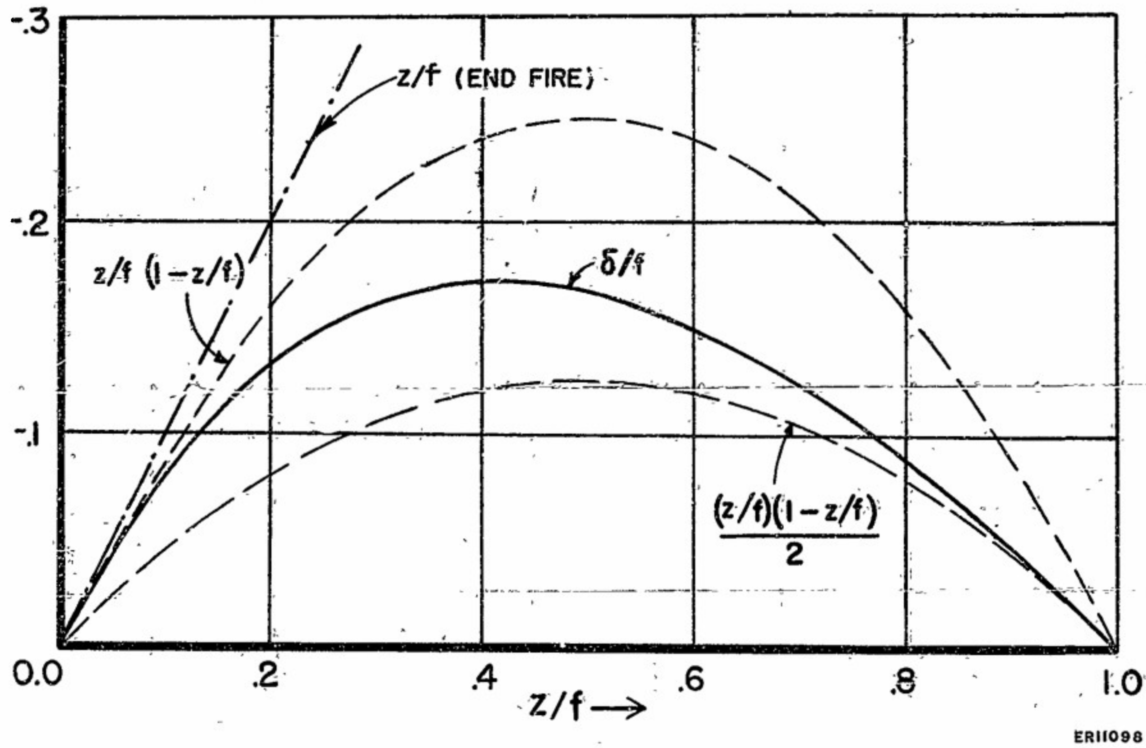

Fig. 2. Path difference δ vs axial distance z.

1.4. RELATIONSHIP BETWEEN APERTURE AND LENGTH OF LINE SOURCE

Consider the distance of a portion of the reflector from the axis

$$r = R \sin\psi \quad \text{or} \quad \sin\psi = r/2f \; . \tag{4}$$

Since $\cos\psi = f/(f + z)$, we may write

$$\left[\frac{r}{2f}\right]^2 + \left[\frac{f}{f + z}\right]^2 = 1 \; . \tag{5}$$

It follows that

$$\frac{z}{f} = \frac{2}{\sqrt{4 - (r/f)^2}} - 1 \; . \tag{6}$$

This is shown in Fig. 3. The series expansion for the function starts off as

$$\frac{z}{f} = \frac{1}{8}\left[\frac{r}{f}\right]^2 + \frac{3}{128}\left[\frac{r}{f}\right]^4 + \cdots\cdots \; . \tag{7}$$

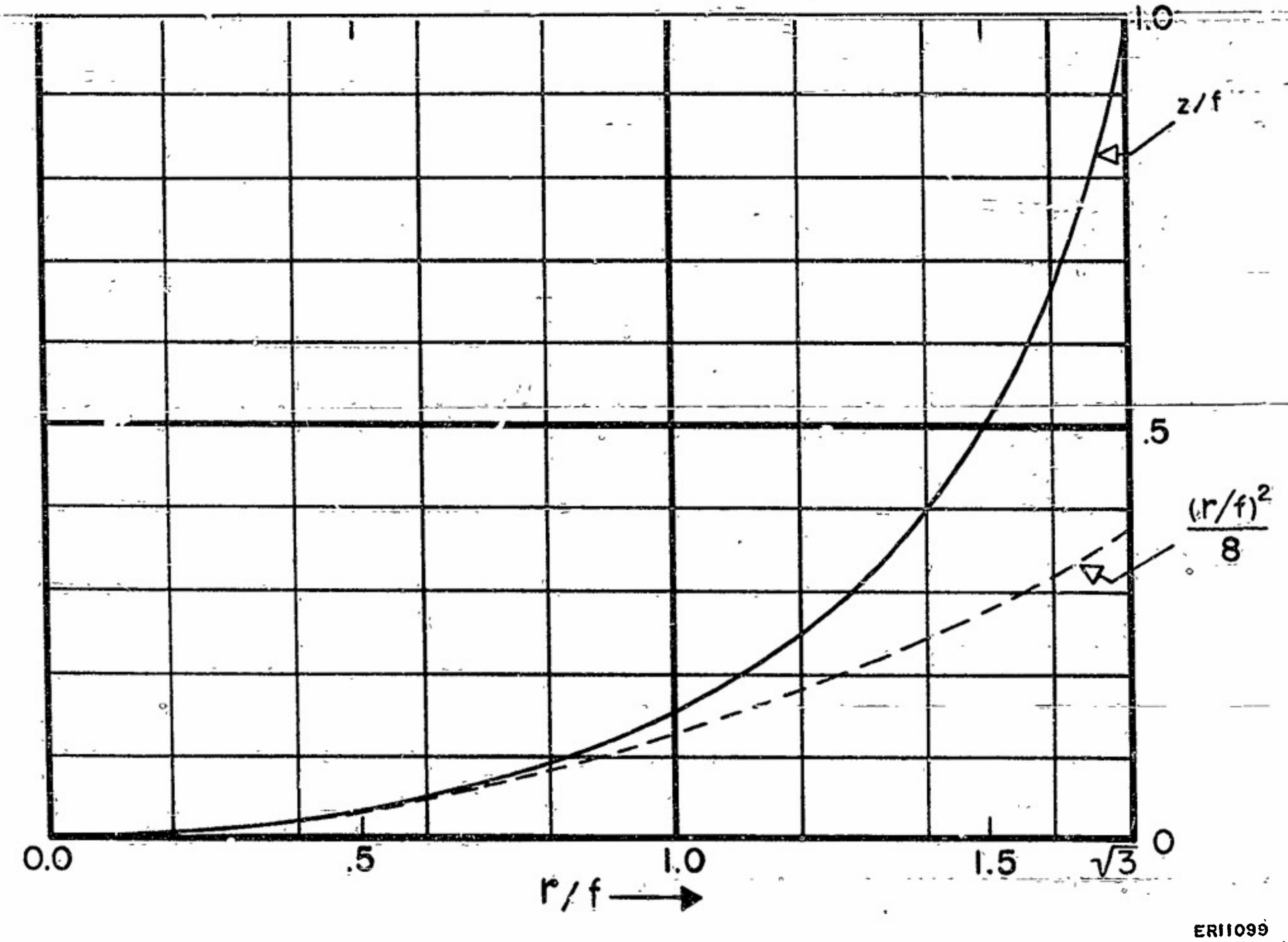

Fig. 3. Feed length z vs aperture radius r.

2. PHASE AND AMPLITUDE DISTRIBUTION ON THE LINE SOURCE

2.1. GENERAL REQUIREMENTS FOR PHASING

The line source can be designed on the basis of Eq. (1) or (3), each of which requires a non-constant phase gradient along the source. In terms of Eq. (1), for example, the line source must be such that it radiates at an angle $2\psi = 0$ at $z = 0$, is broadside at $z/f = \sqrt{2} - 1$ and radiates at an angle $2\psi = 120°$ when $z = f$. In terms of Eq. (3) we let

$$\delta = z - \frac{2fz}{f + z} = \int_z^0 \left[\frac{2f^2}{(f + z)^2} - 1\right] dz = \int_z^0 \frac{\lambda}{\lambda_g} dz \qquad (8)$$

where λ_g can be controlled by loading.

Under these conditions, the illumination on the sphere would be properly phased except for the difference between Fresnel diffraction theory and geometric optics. Actually, there are so few wavelengths in any section of the line source that the illumination spreads out by diffraction. In other words, the amplitude and phase at any point on the sphere is a Fresnel integration of contributions from the entire line source. To date, this integration has not been carried out. A still more rigorous solution, of course, would be the solution of the electromagnetic boundary conditions.

In passing, it should be mentioned that one way to improve upon a point source is to use a short end-fire array, such as a polyrod, in the region near $z = 0$. It is possible to estimate the length of an end-fire array that can be used for a tolerable phase error. Let the greatest phase error be $\epsilon = \lambda/16$. The phase shift along an end-fire array is $\alpha = -z$ (see Fig. 2). But the required phase is $\delta = z - 2fz/(f + z)$. Now let $\delta - \alpha = \epsilon$. Combining and solving for z, we get the maximum length

$$\ell = \frac{1}{4}(\epsilon + \sqrt{\epsilon^2 + 8\epsilon f}) \approx \sqrt{\epsilon f/2} \quad . \tag{9}$$

This means that at X-band an end-fire array in a reflector of 30-inch radius could extend about 0.3 inch. Experimental tests with a polyrod verified the existence of an upper limit at about this length. For lengths greater than 1.2 inches, the pattern deteriorated badly.

2.2. GENERAL REQUIREMENTS FOR AMPLITUDE

The amplitude distribution along the line source can be determined directly on the basis of Eq. (5). The projected area A of a spherical cap on a plane normal to the axis is given by

$$A = \pi r^2 \quad . \tag{10}$$

The illuminated area of the spherical cap, projected on this plane, will hereafter be called the aperture. This is the plane shown in Fig. 4(a).

Combining Eqs. (5) and (10) yields

$$\frac{1}{4\pi}\frac{A}{f^2} + \frac{f^2}{(f + z)^2} = 1 \quad . \tag{11}$$

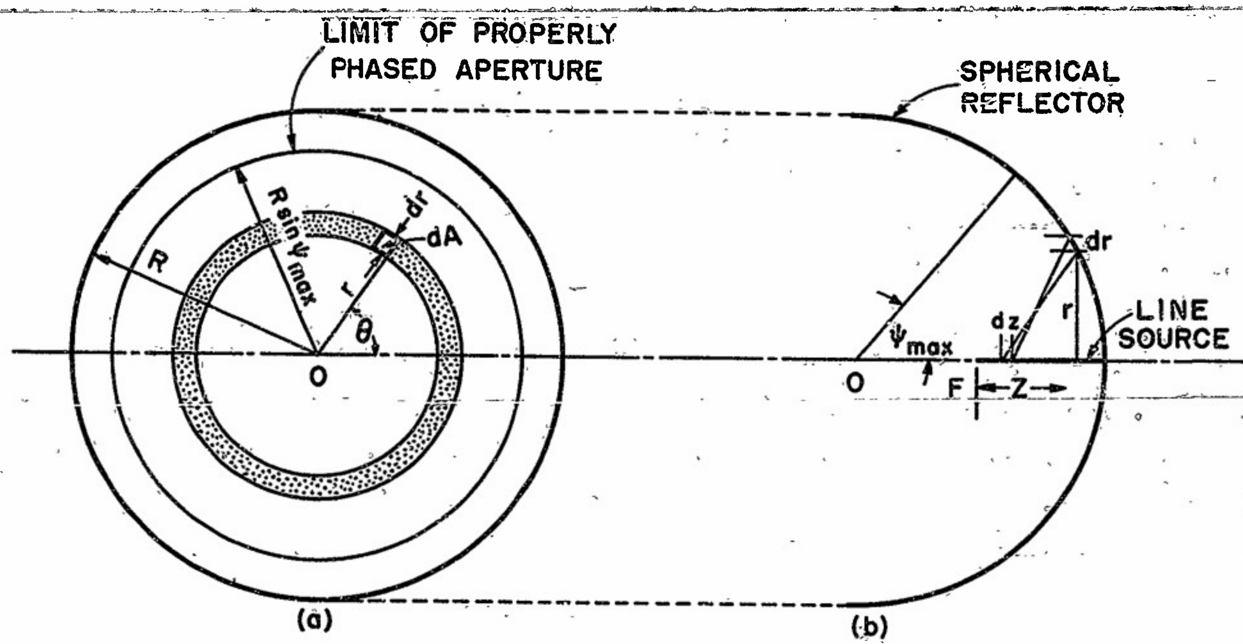

Fig. 4. Coordinate system for aperture illumination.
(a) *Front view* of a hemispherical reflector showing projected surface areas.
(b) *Side view* showing a cross section through axis.

On differentiating, we obtain

$$\frac{dA}{4\pi f^2} = \frac{2f^2}{(f + z)^3}\ dz \ . \tag{12}$$

With the aid of Eq. (12), the intensity distribution on the line source can be related to the desired intensity distribution across the aperture. If the approximations of geometric optics are used, concentric annular areas (one of which is represented by the shaded area in Fig. 4(a)) are illuminated by each element of length dz. Assume the desired aperture power distribution to be represented in polar coordinates by

$$\frac{dP}{dA} = \left[\frac{dP}{dA}\right]_{r_0,\theta_0} F_1(r)F_2(\theta), \tag{13}$$

where r and θ are as shown in Fig. 4(a). Note that r, θ and ψ do not form the usual spherical coordinate system. One can select F_1 for optimum pattern characteristics while F_2 is determined entirely by the patterns of the individual elements of the line source (whether continuous or discrete) in the plane which passes through the element and is transverse to the axis. Note that if F_2 is a constant, a pencil beam with side lobes similar in all planes will result. However, F_2 is not readily adjustable. For example, with dipoles

$$F_2(\theta) \approx \frac{\cos^2(\pi/2 \cos \theta)}{\sin^2 \theta} .$$

If we confine the discussion to radiating elements fed from a common transmission line, then for dipoles a large portion of even this pattern will be shadowed by the transmission line. The same will be true for slots. This immediately indicates that it is desirable to consider both an illumination which is restricted to one side of the line source, and a more symmetrical distribution illuminating the whole cap.

Figures 4-6, along with elementary geometrical considerations, reveal the following: If the illumination is on both sides of the line source, then the optimum use of the reflector is obtained when the reflector is a hemispherical cap. In this instance, the total angle of scan realizable is $180° - 2\psi_{max}$ where $2\psi_{max}$ is the angle subtended at the center by that portion of the area of the hemisphere which is illuminated by the feed. If one wishes to illuminate the maximum possible area, then $2\psi_{max}$ is 120° and the total scanning angle is 60°. (This is the case illustrated in Fig. 5.) A larger scan can be obtained by cutting down the angular spread of the illumination; if $2\psi_{max}$ is 60°, i.e., if the illumination extends ± 30° on either side of the source, then a 120° scan can be obtained. Now, as stated previously, the effective aperture of the system for the purpose of determining beam width is the projection of the illuminated area on a plane perpendicular to the axis of the system; hence, the effective length of aperture for illumination on both sides of the source is $2R \sin \psi_{max}$. At the same time, the projected aperture length of the entire reflector (the illuminated portion plus dead space) is $2R$, and the ratio of effective aperture length to the total length is $(2R \sin \psi_{max})/2R = \sin \psi_{max}$.

If the illumination is on only one side of the line source (Fig. 6) and covers an angle ψ_{max}, the reflector can be cut down to subtend an angle $180° - \psi_{max}$ without decreasing the scanning range, and the total angle of scan realizable is again $180° - 2\psi_{max}$. The effective aperture length is now $R \sin \psi_{max}$, just half what it was with the illumination on both sides of the feed. The total available projected aperture length (including dead

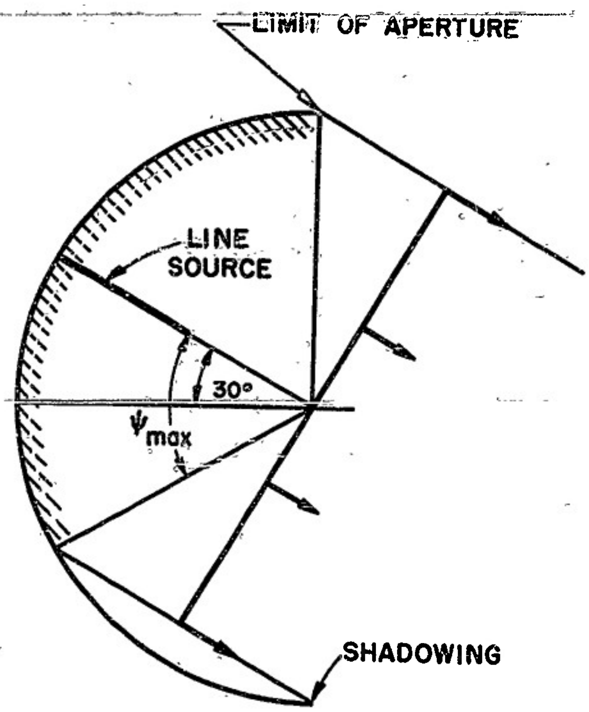

Fig. 5. Scanning limitation--full illumination (illuminated area hatched).

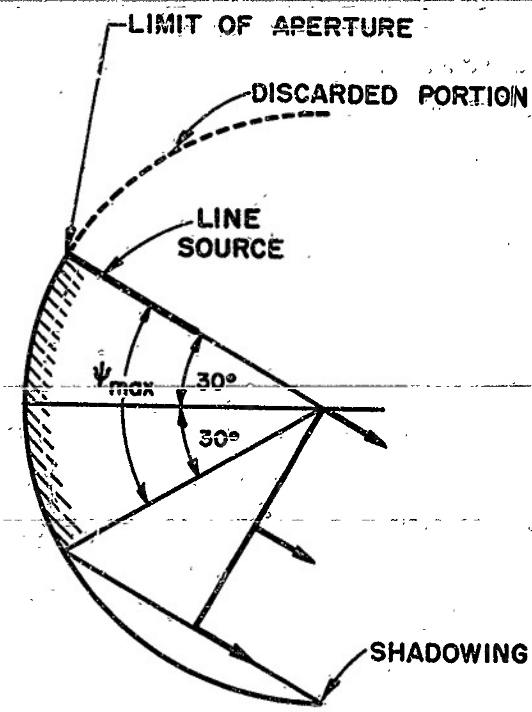

Fig. 6. Scanning limitation--illumination on one side of source (illuminated area hatched).

space) is $2R\cos(\psi_{max}/2)$. The ratio of effective aperture length to total length is $\sin(\psi_{max}/2)$.

If we assign a figure of merit on the basis of these ratios of effective to total available aperture for the two types of illumination, the ratio of the figures of merit is $2\cos(\psi_{max}/2)$ in favor of the first type. Thus, in all cases, for a given scan, illumination on both sides of the feed makes more effective use of the reflector area by involving less dead space. This, of course, presupposes that this type of illumination can be secured without making the line source unduly complicated. A method of doing this is described briefly at the end of this paper.

If we return now to Eq. (13), and assume F_1 and F_2 to be independent, integration gives

$$P = \left[\frac{dP}{dA}\right]_{r_0,\theta_0} \int_0^{x} F_1(r) r \, dr \int_0^{2\pi} F_2(\theta) \, d\theta \; . \qquad (14)$$

The second integral yields only a multiplicative constant which we shall call C. Consequently, the mathematics which follows will apply to both the off-axis and the symmetrical types of illumination.

Combining Eqs. (1) and (4), we have

$$r = R\sqrt{1 - \frac{1}{(1 + z/f)^2}} \quad .$$

From Eqs. (10) and (12)

$$\frac{r\,dr}{dz} = \frac{4f}{(1 + z/f)^3} \quad .$$

Then, from the above,

$$P = BC\int_0^z F_1\left[R\sqrt{1 - \frac{1}{(1 + z/f)^2}}\right]\frac{4f}{(1 + z/f)^3}\,dz \quad . \qquad (15)$$

Hence,

$$\frac{dP}{dz} = 4BC\,\frac{F_1(z)}{(1 + z/f)^3}\,f \quad . \qquad (16)$$

Here, dP is the power radiated per length dz of line source. The constant B represents $[dP/dA]_{r_0, \theta_0}$, and is determined entirely by the input excitation level of the feeding transmission line. Hence B and C can be combined to form one constant D.

Now let P_z be the power level at an arbitrary point z on the transmission line. Then[2]

$$dP_z = -W\Phi(z)dz \quad , \qquad (17)$$

where $\Phi(z)$ is dP/dz as just derived, and W is a normalization factor relating the power in the transmission line to the power radiated. If we let the input power to the line be 1,

$$P_z = 1 - W\int_0^z \Phi(z)dz \quad . \qquad (18)$$

[2]Cf. Randal McG. Robertson, "Variable Width Waveguide Scanners for Eagle (AN/APQ-7) and GCA (AN/MPN-1)," Radiation Laboratory Report No. 840, Massachusetts Institute of Technology, Cambridge, Mass., 1946.

Finally, if g is the power to be delivered to the matched load* at the end of the array, then

$$W = \frac{1 - g}{\int_0^L \Phi(z)dz} \tag{19}$$

where L is the total length of the array. Then, the coupling coefficient for a unit length is

$$K(z) = \frac{dP_z}{dz} \cdot \frac{1}{P_z} = \frac{W \cdot \Phi(z)}{1 - W\int_0^z \Phi(z)dz} . \tag{20}$$

One convenient way of illuminating only one-half of the spherical cap is to use radiators on a rectangular waveguide that is so loaded as to give the required phasing. The radiators, as has been indicated, can be slots or dipoles. The loading can be accomplished in many ways, e.g., by corrugations, by ridging, by dielectric or by varying the wide dimension of the guide (constriction loading). The sources actually built used dielectric and constriction loading.

As usual, for low side lobes the greatest intensity of illumination should be at the center of the illuminated part of the reflector, which, for this type of feed, is now in the top half of the spherical cap. This means that F_1 in Eq. (13) should be, say, sin $(\pi r/r_m)$, where r_m represents the value of r at the extremity of the aperture. With this kind of feed, fortunately, transverse slots or dipoles produce a more or less symmetric illumination of the aperture in both E- and H-planes because their own patterns fall off to zero along their lengths.

2.3. TWO EXPERIMENTAL FEEDS

The two feeds about to be described were built on the above hypotheses, and were designed to feed a spherical cap of 30-inch radius. One is a dipole array in a constriction and polystyrene loaded waveguide, the other is a slot array in a polystyrene loaded guide. The former extends to $z/f = 0.414$, thus illuminating 45° of the total possible 60°, and the latter extends to $z/f = 11/15$, illuminating 54° of a possible 60°. These feeds are shown in Fig. 7.

*Since resonant arrays are not readily adaptable to the requirements of a non-constant phase gradient, such as is necessary here, travelling wave arrays must be used. These, of course, require the absorption of a certain amount of power at their termination.

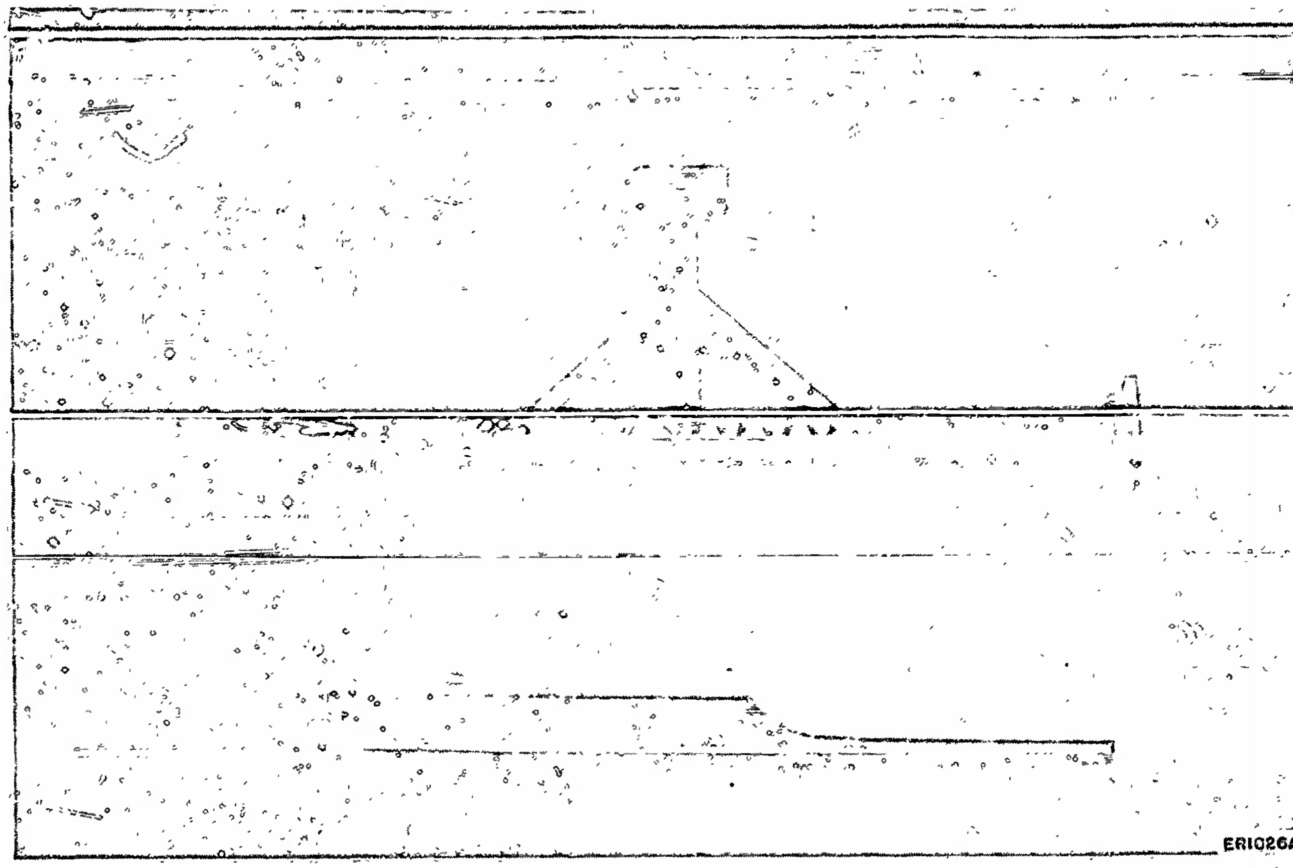

Fig. 7. Experimental line sources.

3. THE DIELECTRIC LOADED SLOT ARRAY

3.1. GENERAL CONSIDERATIONS CONCERNING SLOTS IN A WAVEGUIDE

For the present purpose, slots cut in the narrow side of a rectangular guide were chosen; this made milling much simpler,[3] since such slots must be extended onto the broad side to reach resonant length.

Slots of this type are shunt-coupled to the guide. Their conductance increases from zero with the angle of tilt (measured from the transverse direction). With these slots, phase reversal can be obtained by changing the sense of the angle. This allows a closer spacing of the radiating elements to give a smooth variation in phase along the guide. The tilting is a disadvantage, since it introduces cross-polarization components in addition to those inherent in the geometry of the system. However, even for a small sphere of the dimensions used in the experimental work for this paper, the amount of tilt per slot is not,

[3] W. H. Watson, *The Physical Principles of Wave Guide Transmission and Antenna Systems*, Oxford; 1947.

in general, prohibitive; for larger radii the tilt would become smaller, since the power to be extracted per unit length becomes smaller with increasing radius.

3.2. RELATION BETWEEN LOADING AND THE REQUIRED PHASE

The direction of radiation from a linear travelling wave slot array in a waveguide, with phase reversal between alternate slots, is given in terms of the notation used here by the formula

$$\cos 2\psi = \lambda/\lambda_g + (N - 1/2)\lambda/s, \quad N = 0, \pm 1, \pm 2, \ldots \tag{21}$$

where λ_g is the guide wavelength and s is the slot spacing. If, for a given λ, λ_g and s, more than one N can be chosen to give a real value for $\cos 2\psi$, then the array will have principal maxima of radiation in more than one direction. Hence the parameters must be chosen in such a way that 2ψ is unique. Once the operating wavelength is set, we have at our disposal two parameters, λ_g (which can be controlled by guide loading) and s. The procedure now is to compute the desired 2ψ for a given z on the array. For a given s, this will define the value of λ_g necessary at the point z, and hence determine the loading at that point.

3.3. DIELECTRIC LOADING

The simplest types of dielectric loading are those in which a given fraction of the guide is filled with a dielectric whose boundaries are parallel to the guide walls. If the guide is to be slotted on the narrow side, complications are avoided by having the dielectric-to-air interface parallel to the narrow side, the dielectric extending completely from top to bottom of the guide. Then the slots will be backed either entirely by dielectric or entirely by air.

The theory of a rectangular waveguide partially filled with dielectric has been discussed by Pincherle[4] and Frank.[5] On the basis of this theory, a curve of λ_g vs thickness of dielectric was prepared for the case described above, using a dielectric constant of 2.50 (the measured value of ϵ for the polystyrene used). By combining this curve with one drawn for λ_g vs z, as computed above, we arrive at an actual design curve of dielectric thickness against distance down the line source.

[4] L. Pincherle, "Electromagnetic Waves in Metal Tubes Filled Longitudinally with Two Dielectrics," *Phys. Rev.* 66, 118-130, (1944).

[5] N. H. Frank, "Waveguide Handbook, Section V, Dielectric Structures in Waveguides," Report T-9, Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass., 1942.

3.4. DESCRIPTION OF THE ARRAY

The possible range of λ/λ_g at 3.3 cm, using polystyrene in standard *X*-band guide, is from 0.692 to 1.406. Careful consideration of Eq. (21) shows that, with this limited range of variation in combination with proper choice of slot spacing, a polystyrene loaded waveguide array with *constant* phase gradient can be made to radiate in any given direction from 0° to 180° with only one principal maximum. Presumably, then, an array with non-uniform loading and non-uniform slot spacing, fed at both ends or at some point between the ends, could be made to radiate over the spread of angles represented by the variation of 2ψ from 0° to 120°.

Calculations based on the possible ranges of the parameters verify that this is so; indeed, it turns out that the variation can be accomplished (except for a negligible 5° near $z = f$) by using two constant slot spacings--one for the section of the line source on each side of a single feed point. The arrangement of dielectric settled upon is shown in Fig. 8. The feed point is at the point of maximum thickness of dielectric. The section from $z = 0$ to $z/f = 0.25$ has a slot spacing of 0.384 inch. The other section has a slot spacing of 0.580 inch. Since each section constitutes a travelling wave array, matched loads are used at the ends. The entire arrangement is made clear in Fig. 7 (center). The waveguide which feeds power into the slotted guide is bifurcated by a septum which divides the power between the two sections in the ratio required by the amplitude distribution about to be discussed. A screw inserted in the septated region proved sufficient to bring the two sections of the feed into proper phase relationship.

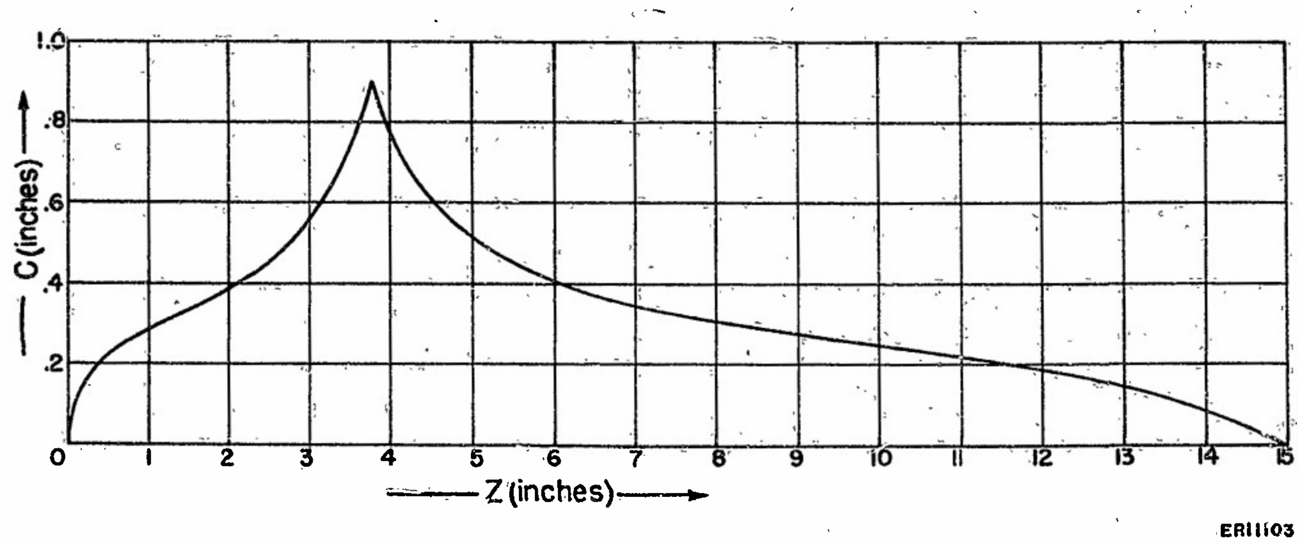

Fig. 8. Thickness of polystyrene loading vs z.

3.5. REGULATION OF AMPLITUDE

The fraction of power which should be extracted from the guide by each slot was computed according to Eq. (20) to give a sinusoidal distribution of intensity on the energized portion of the reflector. It remained to express this information in terms of slot parameters. This had to be done empirically, since slot conductance for transverse slots, where mutual coupling is very high, is a function not only of free space wavelength and angle of tilt but also of the thickness of the dielectric loading and of the spacing between the slots. In addition, the resonant length of the slots is not the same for loaded guide as it is for empty guide. An expression for slot conductance as a function of dielectric thickness was derived on the basis that slot conductance is proportional to the square of the current density in the side wall of the guide. It is plotted in Fig. 9. This expression is only qualitatively correct since, perforce, it ignores mutual coupling. But it does indicate that, for dielectric thickness up to over half the width of the guide, the conductance of slots on the dielectric side is increased over that in empty guide, the maximum increase occurring when the dielectric thickness is about one-fourth of the guide width. After that point, the relative conductance decreases to a value about one-half that of empty guide. The reverse happens for slots on the air side. These indications were borne out experimentally.

Since, on the average, the thickness of the dielectric (see Fig. 8) is less than half the width of the guide, the slots were located on the dielectric side. This gave the advantage of increased conductance, so that it was possible to base the computations on a total loss of 10% in the matched loads. The measured loss was 14½%.

4. THE DIPOLE ARRAY

4.1. GENERAL CONSIDERATIONS

In a uniform travelling wave array with similar radiating elements sufficiently close together, the angle of the principal maximum is given by (cf. Eq.(21))

$$\cos 2\psi = \lambda/\lambda_g . \tag{22}$$

In a rectangular guide operating in the TE_{10} mode

$$\lambda/\lambda_g = \sqrt{\epsilon_r - (\lambda/2a)^2 + \beta} , \tag{23}$$

where ϵ_r is the relative dielectric constant, a is the guide width and β represents the effect of the dipole loading on the phase velocity. For an array of resonant dipoles β is presumed small; it is, however, appreciable for the large couplings used. By ray or path length analysis, as carried out before, the required phase velocity along the line source is found to be given by the following functions:

$$V/V_g = -\frac{\lambda}{\lambda_g} = \frac{d\ \delta(z)}{dz} = 1 - \frac{2f^2}{(f+z)^2} \ . \tag{24}$$

Combining Eqs. (23) and (24) and using $\epsilon_r = 1$ and $\beta = 0$ yields an explicit relation for guide width as a function of z:

$$a/\lambda = \frac{1}{2}\left[1 - \left|\frac{2}{(1+z/f)^2} - 1\right|^2\right]^{-1/2} \ . \tag{25}$$

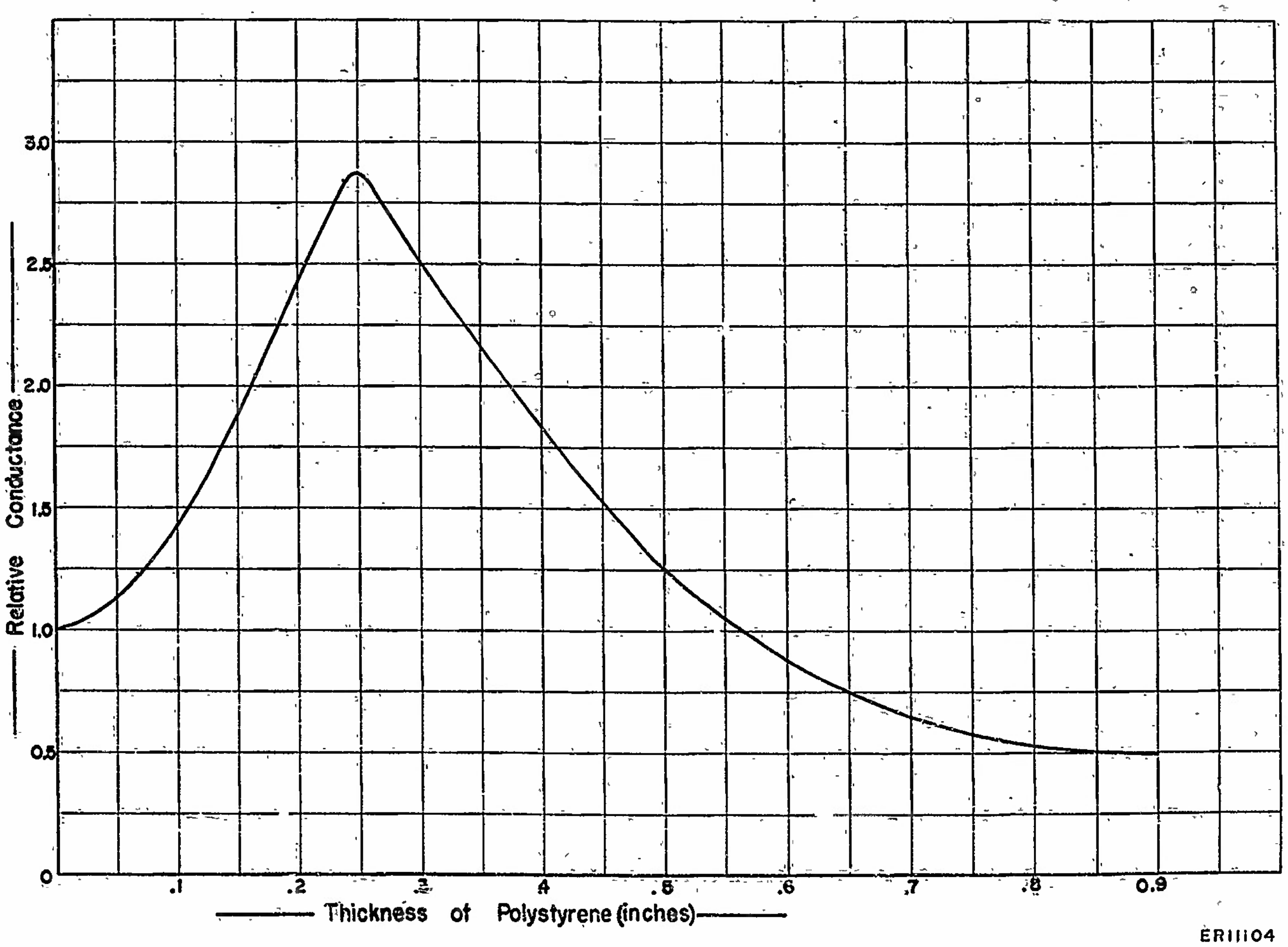

Fig. 9. Slot conductance vs loading thickness.

4.2. DESCRIPTION OF THE DIPOLE ARRAY

To obtain the needed phase characteristics, X-band guide (λ = 3.3 cm) was loaded by varying a according to Eq. (25). This equation is plotted in Fig. 10. In order to obtain the proper phasing in the region near z = 0 it was necessary to load the first portion of the line source (up to z/f = 0.87) with polystyrene (see Fig. 11).

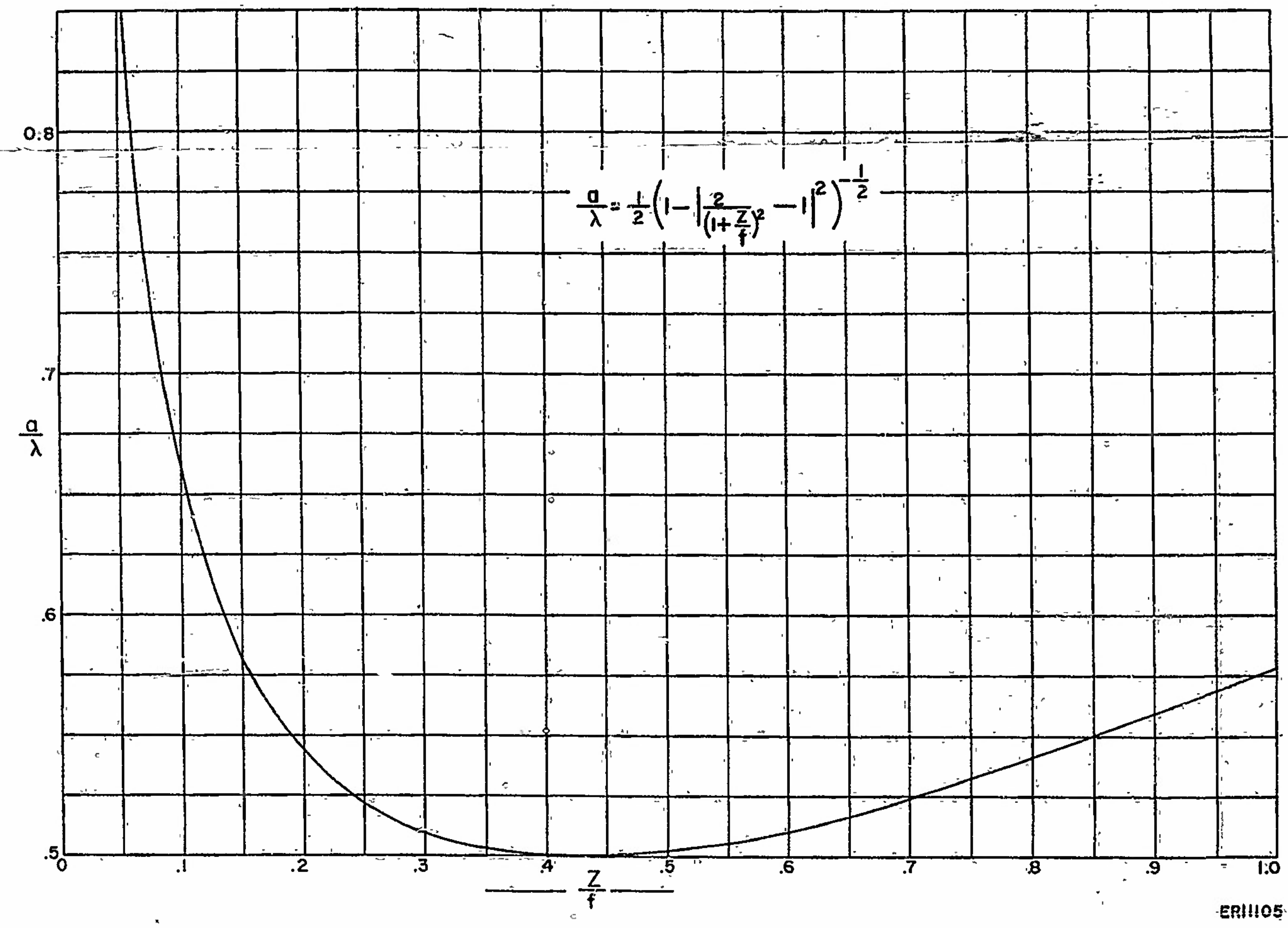

Fig. 10. Wave guide width vs z.

Dipoles on a line 0.52 inch from the straight edge of the guide were placed transverse to the line source and spaced so that each dipole illuminated equal areas of the 45° aperture. The equation for determining this spacing is

$$\frac{z_2}{f} = \frac{R(1 + z_1/f)}{\sqrt{R^2 - 40(1 + z_1/f)^2}} - 1 \quad (26)$$

where z_1 and z_2 represent successive locations on the line source. This spacing was also near optimum for achieving best directivity for each section of the array.[6] Probe settings were adjusted to approximate a sinusoidal power distribution across the aperture. (See Figs. 12 and 13.)

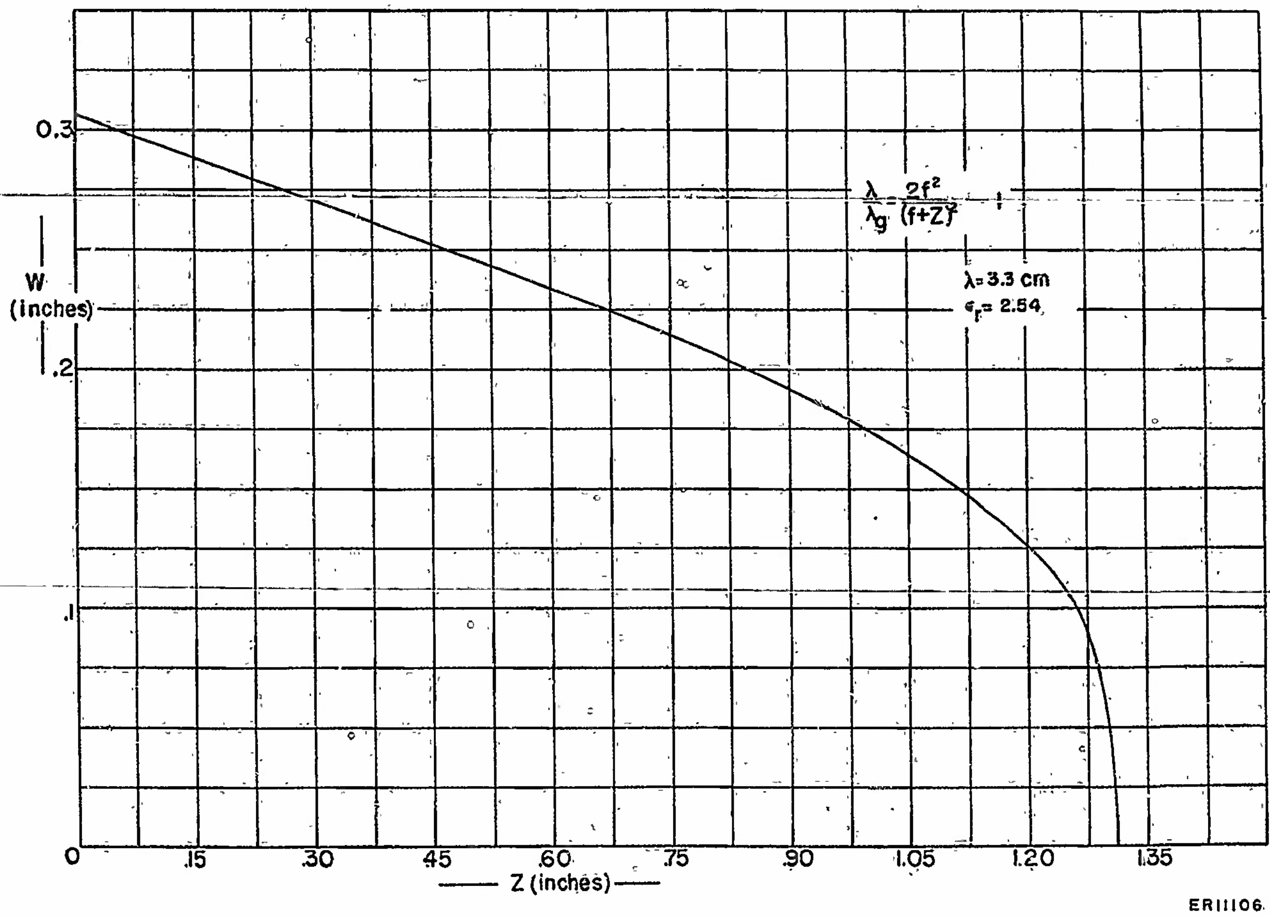

Fig. 11. Polystyrene width vs z (dipole feed).

High mutual coupling between dipoles had to be tolerated in order to space the dipoles more closely together and hence reduce the amount of power extracted by each one.

About one-tenth of the input power was absorbed in a matched resistance card on the end of the feeding guide. The final probe depth adjustments were made experimentally by noting the effect of varying probe depths on the primary patterns (see Fig. 14). Without

[6] R. W. King, H. R. Mimno, A. H. Wing, *Transmission Lines, Antennas and Waveguides*, McGraw-Hill Book Co., New York, 1945.

any matching device, the VSWR looking into the waveguide was less than 1.2.

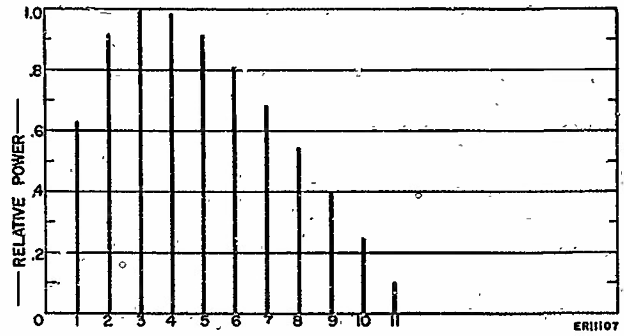

Fig. 12. Radiated power per dipole.

5. EXPERIMENTAL RESULTS

5.1. TESTS ON THE LINE SOURCE

The theoretical primary intensity pattern that should be obtained from the line source when a pick-up horn is rotated in the position of the spherical reflector is shown in Fig. 14. This curve is computed for sinusoidal illumination of the reflector and takes into account the directivity pattern of the horn itself. Representative experimental data are shown on the same figure.

Relative phase of the output of the line sources was also measured by moving a pick-up horn on the same circle. The curves thus obtained indicated phase errors chiefly in regions of low intensity. Fig. 15 is a sample of the experimental data.

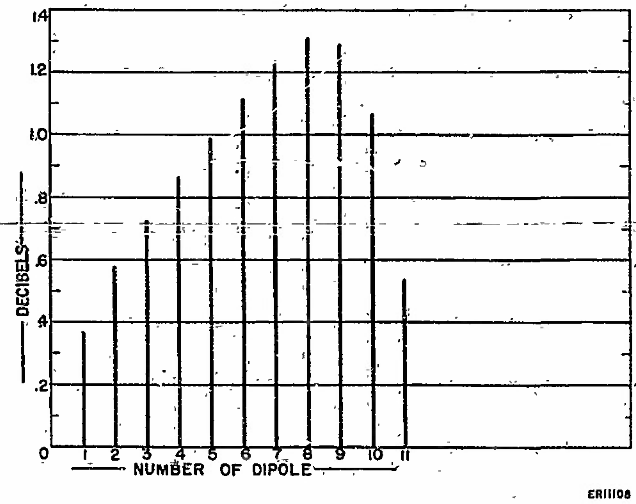

Fig. 13. Decibel drop along feed line per dipole.

5.2. SECONDARY PATTERNS

The reflector used for these experiments had a radius of 30 inches and a total aperture of 54 inches. It is shown in Fig. 16.

In order to make sure that the corrected line sources gave better results than simple horns, a series of *H*-plane patterns was taken, using horns of various sizes. The horn dimensions ranged, in small increments, from plain waveguide to an aperture of 6 inches. The lowest side lobes obtained were 18 db down; the narrowest beam width for that side lobe level was 3°. The narrowest beam width of all the patterns was 2.7° but in that case the side lobes were 15 db down. In addition to the horns, a polyrod 1.2 inches long in the end of a waveguide was tried. The highest side lobes were 24 db down and the *H*-plane beam width was 3½°.

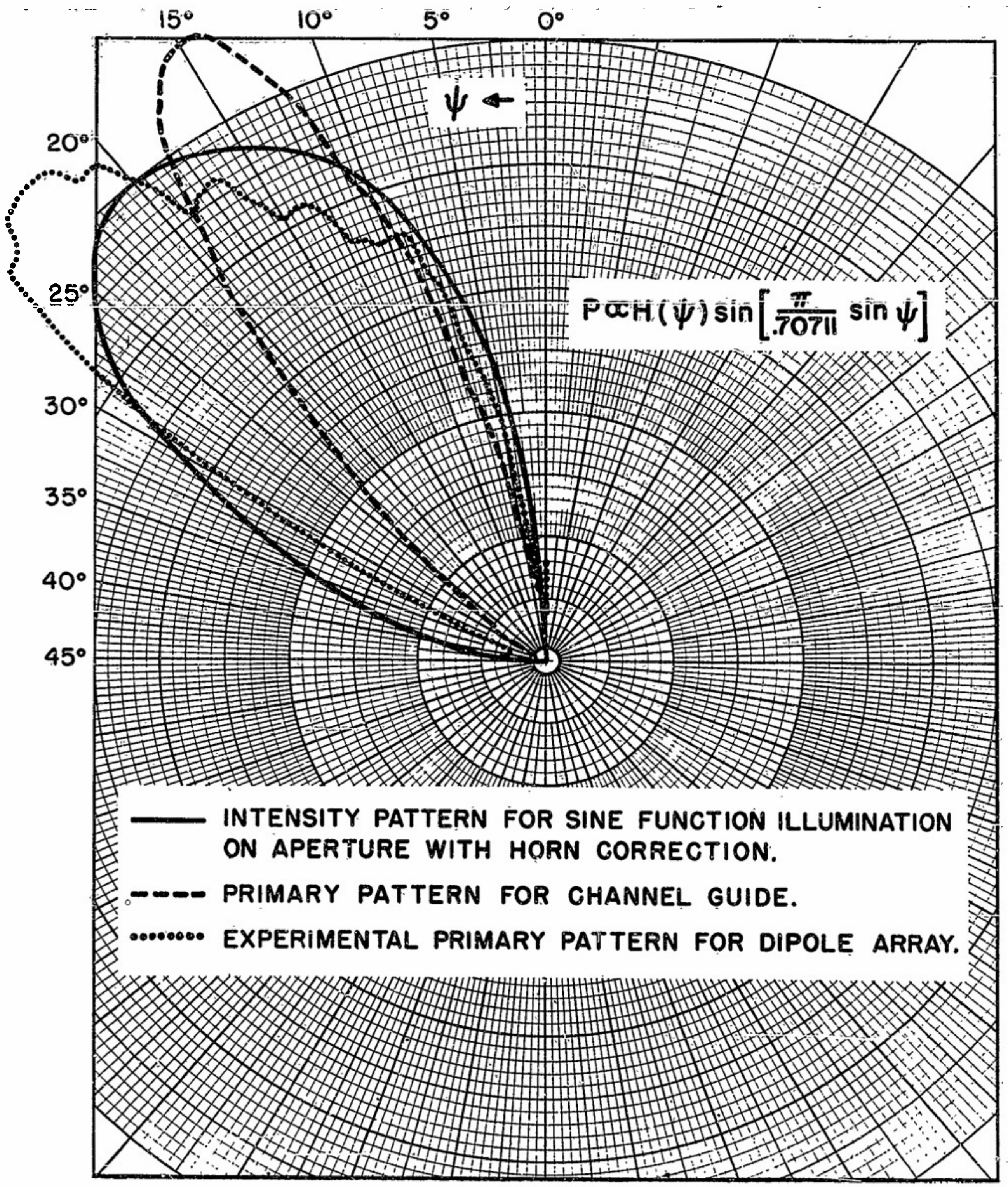

Fig. 14. Primary illumination patterns.

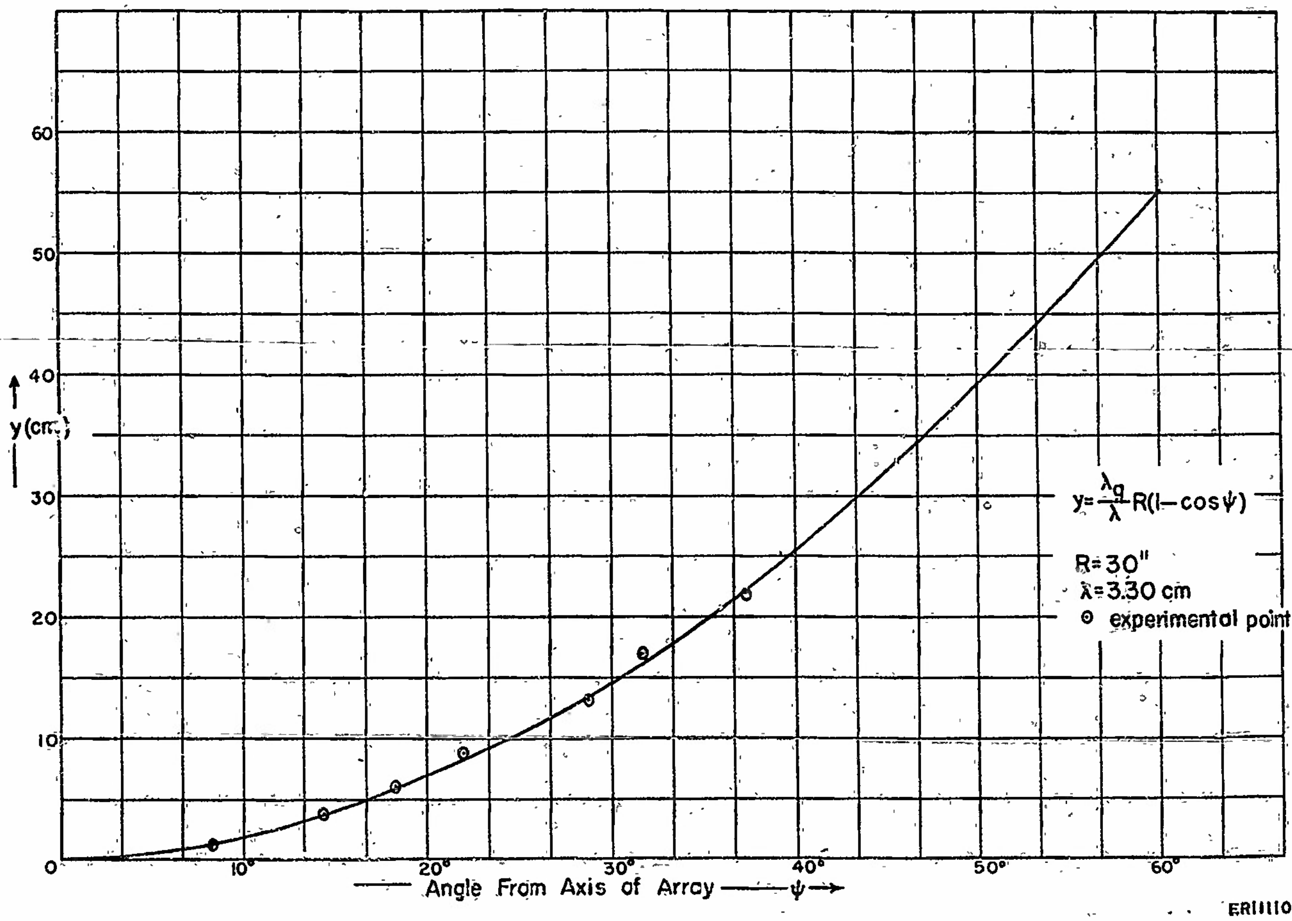

Fig. 15. Phase variation on reflector.

The results with the corrected line sources were as follows: the dipole array gave an H-plane beam width of 3° with 20-db side lobes, and an E-plane beam width of 2.3° with 18-db side lobes (Figs. 17 and 18). The slot array gave an H-plane beam width of 2° with 17-db side lobes (Fig. 19). (The H-plane for the slots corresponds to the E-plane for the dipoles; in this plane, illumination extended on both sides of the line source, whereas in the other plane it was on one side only.) The E-plane pattern of the slot array was not measured because it was known that the length of guide which feeds the line source would lie in a region of high intensity radiation and would spoil the E-plane pattern. This situation could be remedied by making the feeding guide lie flush against the line source; the construction actually used had the advantage of speed in fabrication for meeting a deadline, and the H-plane pattern is considered sufficient to justify the assumptions on which the feed was designed.

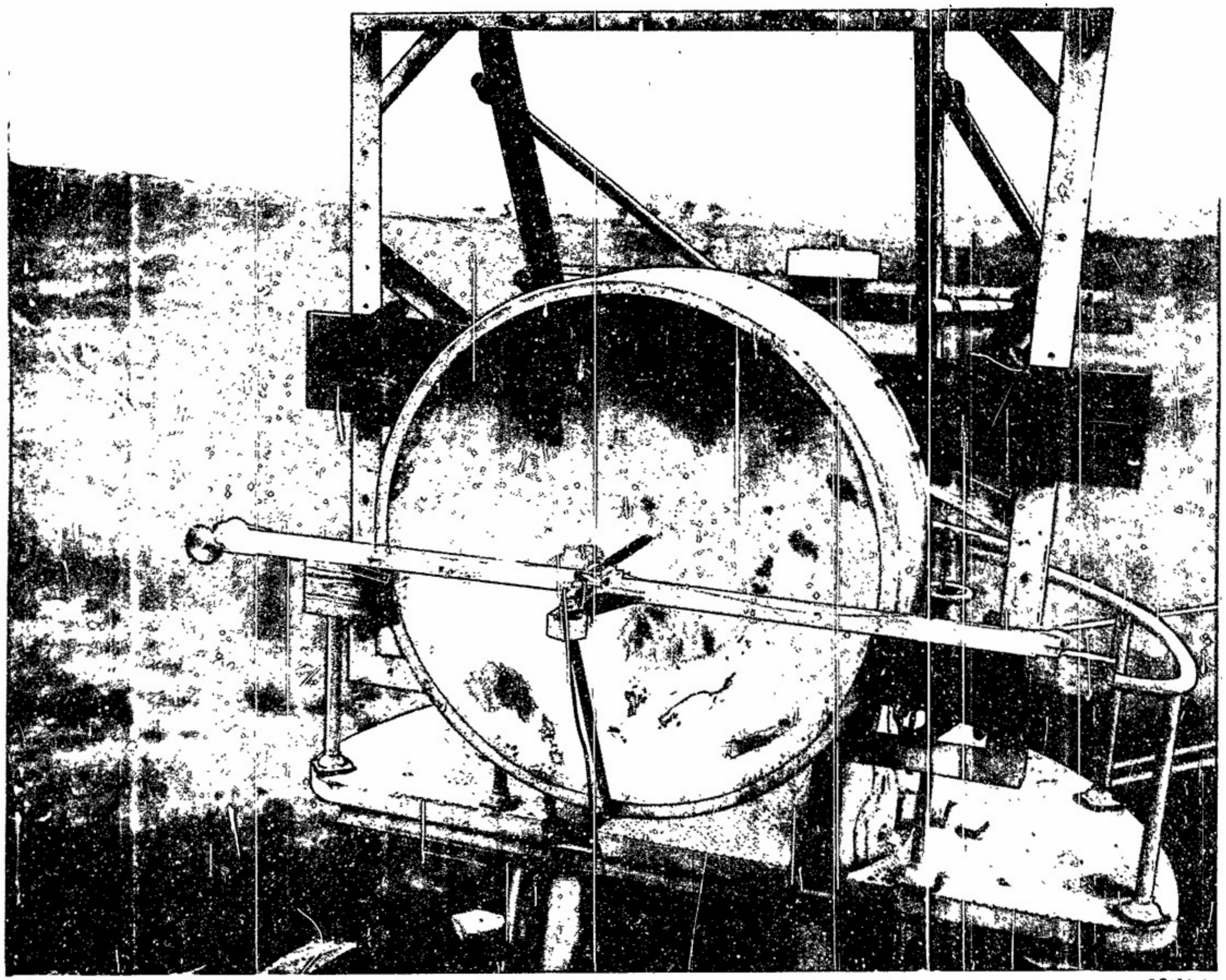

Fig. 16. Experimental reflector.

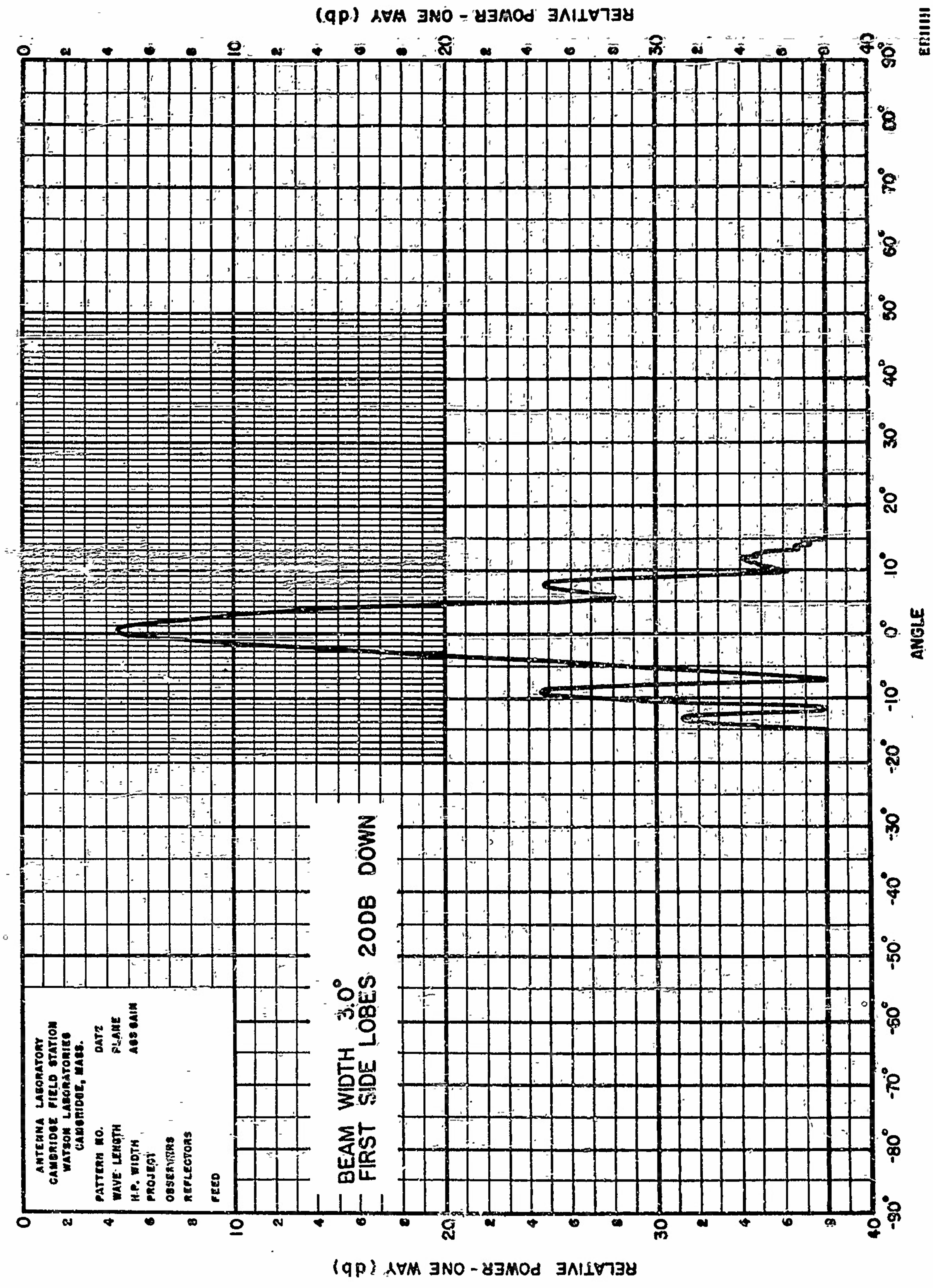

Fig. 17. Dipole feed--*H*-plane pattern.

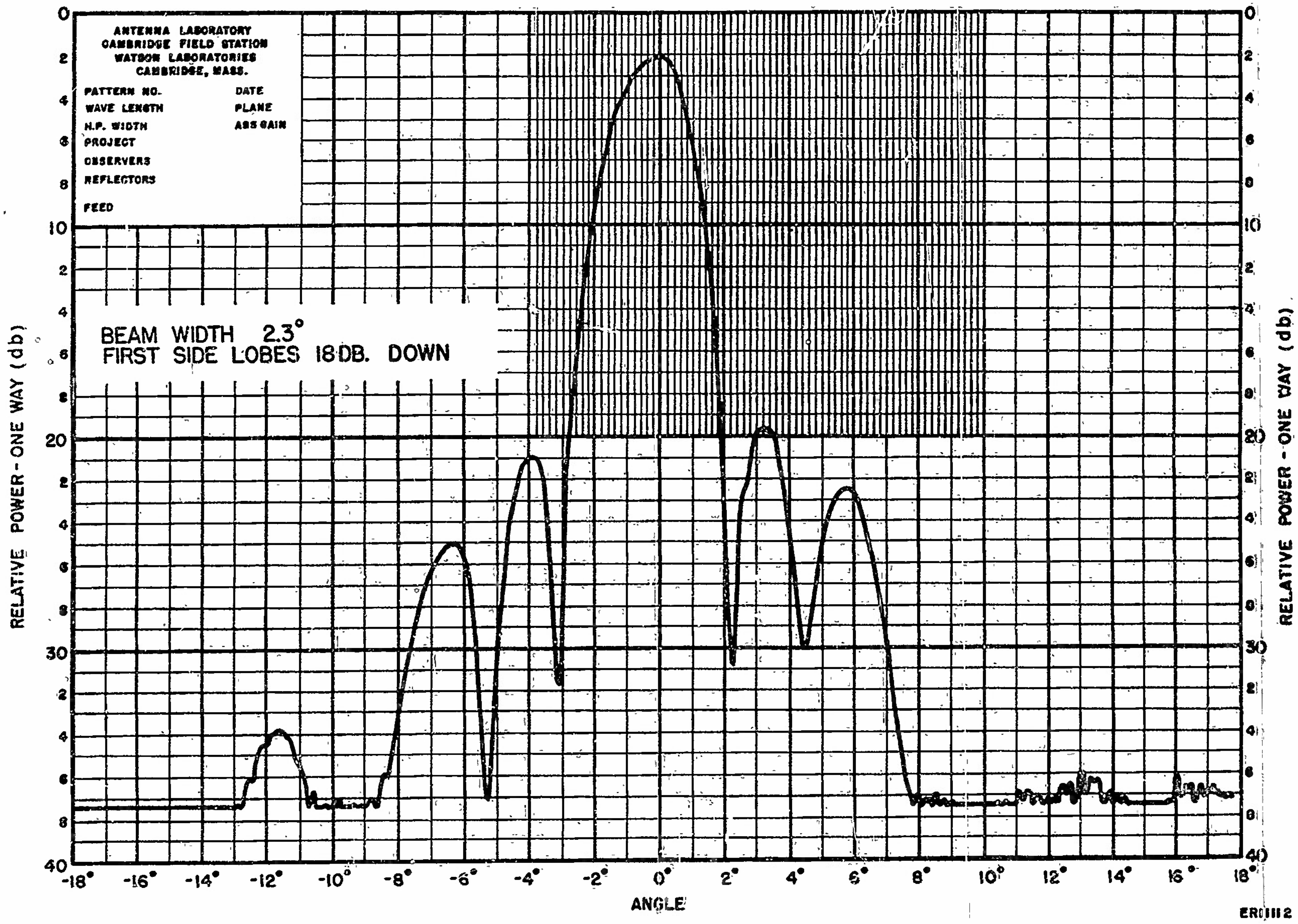

Fig. 18. Dipole feed--*E*-plane pattern.

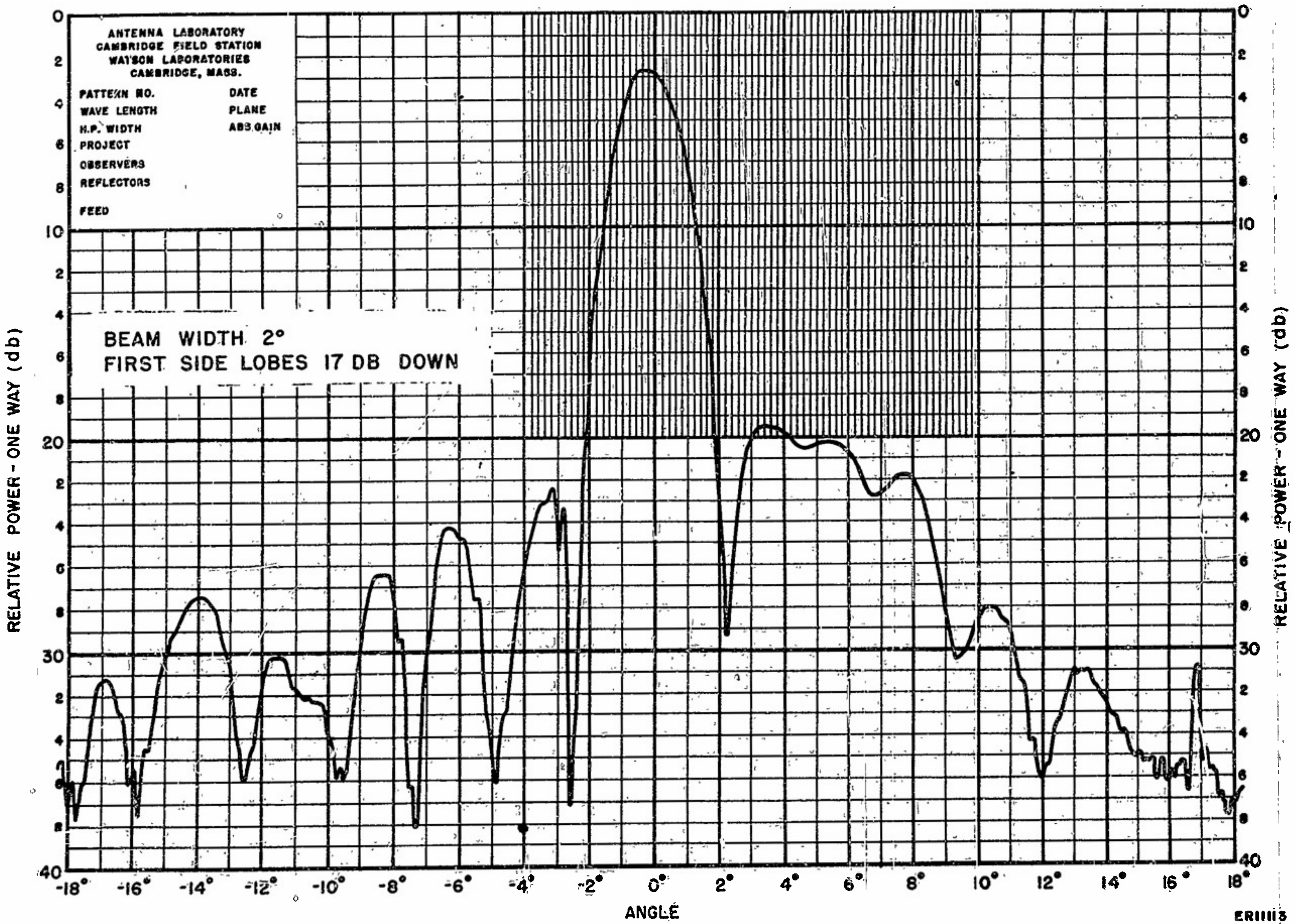

Fig. 19. Slotted feed--*H*-plane pattern.

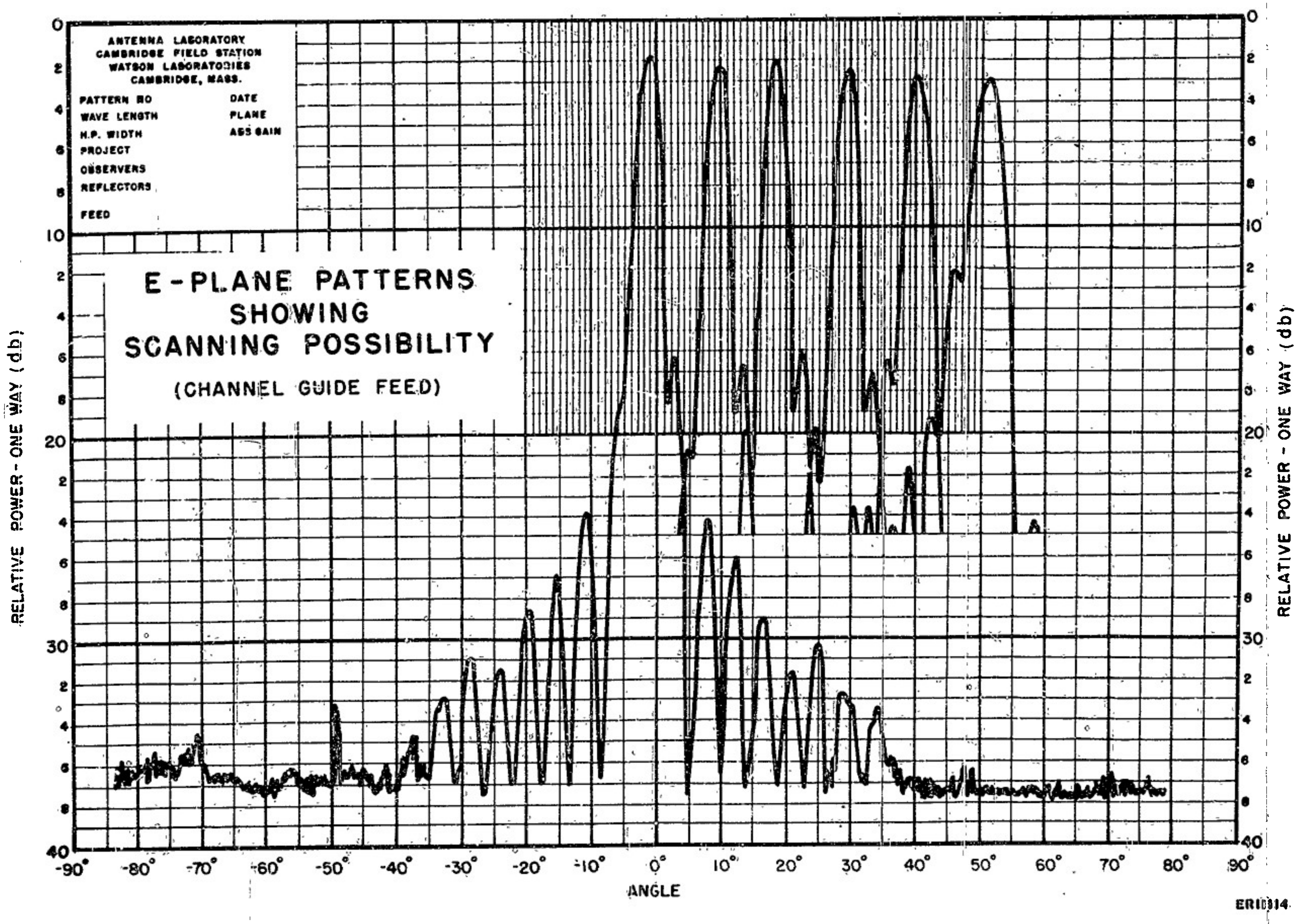

Fig. 20. Experimental scanning patterns.

It will be noted that the side lobe levels obtained with the line sources are about the same as those for the best horns, but that the beam widths are narrower. Since these feeds were the first corrected line sources tried, and no attempt was made to improve the phasing or the amplitude distribution, it is not unreasonable to suppose that the patterns could be substantially improved by taking into account such details as the phase shift introduced by the loading effect of dipoles or slots on the travelling wave in the guide. However, the unimproved feeds are sufficient to vindicate the system.

6. OTHER FEEDS

A very promising type of line source was made by using a continuously radiating channel[7,8] guide (Fig. 7, bottom). This consists of a rectangular X-band guide with one of the narrow walls removed. The phase velocity was controlled by varying the width of the guide. No attempt was made to regulate the amplitude distribution; however, the theory of the channel guide would permit both phase velocity and rate of radiation to be controlled by simultaneously varying the width and the height of the guide. The chief advantages of this feed are: (1) the radiation is effectively coming from the bottom of the channel, which makes it possible to place two channels back to back and illuminate both top and bottom of the spherical cap; (2) construction is mechanically simple. The first advantage is important because analysis shows that a line source off-radius by a distance d effectively shadows a region of area $A' = \pi d^{2/3} R^{4/3}$ rather than $A = \pi d^2$. The use of two dipole or slot arrays back to back would result in one or both of them being substantially off-radius.

The secondary patterns (see Fig. 20) and phase data for the channel guide are better than was expected from the approximate design. The scanning possibility of a line source on a spherical cap is demonstrated by the patterns shown in the same figure.

There have been several other proposals for feeding a half-hemisphere. In 1948, F. J. Zucker of the Antenna Laboratory, AFCRL, worked out surface shapes necessary to produce a corrected line source with (1) equi-phased radiators along a specially curved surface, (2) radiators along a constant phase gradient surface, and (3) a point source illuminating a feeding reflector enclosed in a pillbox. These methods have not been tested experimentally. They have appeared less fruitful because of the size and inertia of the necessary pillboxes and reflectors. However, these shaped surfaces may have superior directivity properties.

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[7] W. Rotman, "The Channel Guide Antenna," *Proceedings of the National Electronics Conference*, vol. V, Chicago, 1950. A more complete treatment is given by the same writer in "The Channel Guide Antenna," AFCRL Report E5054, Cambridge, Mass., January 1950.

[8] Interim Engineering Report on Contract W33-038-ac-16520 (17380), 310-14, Antenna Laboratory, the Ohio State University Research Foundation, pp. 4-1 to 4-16, 1 February 1949.

Air Force Cambridge Research Laboratories E5069
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